A quick polarization document

Start with a very, very quick overview. The polarization of a light field can be described with a four element vector called a stokes vector…(I, Q, U, V)…I is total intensity, Q and U are related to linear polarization, and V is for circular polarization. This stokes vector is transformed with a 4 x 4 matrix called a Mueller matrix. In general V is very small in the natural light field, and we will ignore that here. But the linear algebra looks like:

$$\begin{bmatrix} I' \\ Q' \\ U' \\ V' \end{bmatrix} = \begin{bmatrix} M11 & M12 & M13 & M14 \\ M21 & M22 & M23 & M24 \\ M31 & M32 & M33 & M34 \\ M41 & M42 & M43 & M44 \end{bmatrix} \begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix}$$

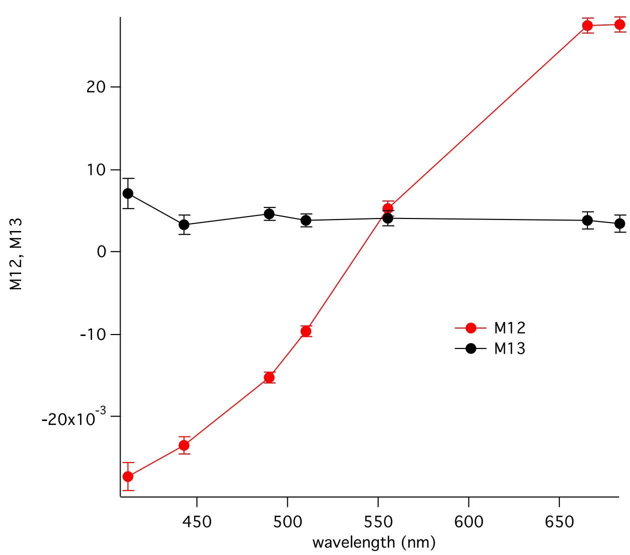
Our detector at the end of the instrument chain just measures the intensity, independent of polarization (or at least we will assume this and group all polarization sensitive things to before this detector). The trick then is to know the first row of the Mueller matrix of the instrument

With the data that Mike F. took, we can fit the data for each wavelength to the equation (note the data I saw already was normalized so I was very close to one).

Signal = M11+ M12 cos(2*angle) +M13 sin(2*angle) )

Where angle is the polarizer angle and I, M12, and M13 are the fitted parameters.

I plotted these fits for a subset of wavelengths that Steph sent me:



You can see that M13 is basically zero and M12 switches sign. When M12 and M13 are close to zero (550 nm) there is basically no polarization sensitivity. The switch from positive to negative reflects the phase shift of 90 degrees in where the max/min is as you go from blue to red. If we do this fit every 10 nm, we can get a very good estimate of the polarization sensitivity of the instrument.

When we have the instrument parameters we can predict the effect of polarization on our signal. We can write what we might expect the stokes vector of the light field to look like assuming a simplification that we have Rayleigh like single scattering, with a maximum degree of polarization of DOPmax:

$$\begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix} = \begin{bmatrix} 1 \\ DOP_{max} \dfrac{-\sin(2\psi)^2}{1+\cos(2\psi)^2}\cos(2\theta) \\ DOP_{max} \dfrac{-\sin(2\psi)^2}{1+\cos(2\psi)^2}\sin(2\theta) \\ 0 \end{bmatrix}$$

where ψ is the scattering angle (180- in water solar zenith angle), and θ is the angle between the arm and the solar azimuth.

So when the instrument measures this, the signal from the detector at the end will be:

$$\begin{aligned} signal &= 1 \\ &+ M12 * DOP_{max} \frac{(-\sin(2\psi)^2)}{1+\cos(2\psi)^2}\cos(2\theta) \\ &+ M13 * DOP_{max} \frac{(-\sin(2\psi)^2)}{1+\cos(2\psi)^2}\sin(2\theta) \end{aligned}$$

DOPmax would vary, but it will be in range of 0.5-0.8 at the site (for a quick reference). I will work on figuring out a good value for DOPmax for the MOBY site. I may have to get Art to do some calculations.